Thursday, August 29

Describe how to plot a point on a polar grid.

(What does a negative angle mean?
What does a negative r-value mean?
How are polar coordinates different from rectangular coordinates?)

August 29 - Day 2

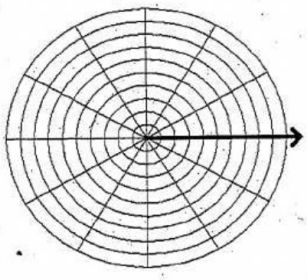
Students will verbally explain how to graph polar functions

(using the words:
angle, radius, terminal, initial...)

Air Traffic Controller

STUDENT ACTIVITY (continued)

As aircraft move through the air space in the vicinity of the control tower, radar tracks their movement by periodically updating their locations. In Question 2(b), you recognized that a distance and an angle are needed to locate an airplane. The **initial ray** for this angle is the ray from the pole O along the positive x-axis, indicated by the dark ray in the polar grid shown below. This ray is called the **polar axis**. The **terminal ray** for an angle that locates the airplane is the ray from the pole passing through the point locating the airplane's position on the radar screen. The angle is determined by rotating the initial ray into the terminal ray. If this rotation is counterclockwise, the angle has a positive measure. If this rotation is clockwise, the angle has a negative measure.



3. On the polar grid above, plot and label the following points.
   a. Point A is located 20 miles from O, at an angle of 60° from the polar axis.
   b. Point B is located 20 miles from O, at an angle of –120° from the polar axis.
   c. Point C is located 15 miles from O, at an angle of 150° from the polar axis.

Air Traffic Controller

STUDENT ACTIVITY (continued)

To locate an airplane on the radar screen, an ordered pair of information is needed, namely a distance from the pole O and an angle from the polar axis. Usually the polar pair for a point on the polar grid is given as (r, θ), where r gives the distance from the pole O and θ gives the angle from the polar axis.

4. Write the polar pairs (r, θ) for the points that were described in the corresponding parts in Question 3.
   a. Point A: $(r, \theta) =$
   b. Point B: $(r, \theta) =$
   c. Point C: $(r, \theta) =$

5. Consider the points (40, 240°) and (40, –120°).
   a. Describe, in words, what occurs when you plot these points.

Same location

   b. A rectangular xy-coordinate grid uses ordered pairs of the form (x, y) to indicate points in the plane. Do the rectangular pairs of the form (x, y) have the same property that you observed for the polar pairs (40, 240°) and (40, –120°)? Justify your explanation.
   c. Name some other polar pairs that identify the points (40, 240°) and (40, –120°). Explain how such other polar pairs can be constructed.

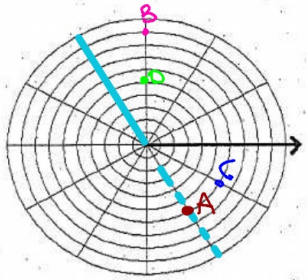
(40, 600°) (40, 960°) (-40, 60)
(40, -480°) (40, -840°)

Air Traffic Controller

STUDENT ACTIVITY (continued)

The r-values in the examples in Questions 3–5 were given as positive values marking the distance of the point from the pole. In general, the r-value in a polar ordered pair (r, θ) is a directed (positive or negative) distance from the pole. For example, if θ is a directed angle, and if $r > 0$, the location of the point on the polar grid is found by moving along the terminal ray, making the angle θ. The value r can be given as a negative number. In this case, the location of the point on the polar grid is found by extending the terminal ray in the opposite (negative) direction, and moving along this opposite extension of the ray, a distance of $|r|$ units from pole O.

6. Plot and label the following polar ordered pairs on the polar grid shown below.
   a. Point A: (–30, 120°)
   b. Point B: (–45, 270°)
   c. Point C: (–30, –210°)
   d. Point D: (–25, –90°)



7. Locate the point given by the polar pair (20, 300°) on the polar grid in Question 6. Next, list three different polar ordered pairs (r, θ) which describe the same point on this grid.

(20, 660°) (20, -60°)
(-20, 120°) (-20, -240°)

Air Traffic Controller

STUDENT ACTIVITY (continued)

8. Points on the polar grid can have multiple polar pairs denoting their location.
   a. Suppose $|r_1| = |r_2|$. Must polar pairs (r_1, θ_1) and (r_2, θ_2) denote the same point on the polar grid? Explain why or why not.

$r_1 = 40$ (40, 150) (-40, 150)
$r_2 = -40$
$r_2 = 40$

   b. Suppose $r_1 = -r_2$, and the polar pairs (r_1, θ_1) and (r_2, θ_2) denote the same point on the polar grid. What can you conclude about the angles θ_1 and θ_2?

(20, 300°)
(-20, 120°)

   c. Suppose the polar pairs (r, θ_1) and (r, θ_2) denote the same point on the polar grid. What can you conclude about the angles θ_1 and θ_2?

(20, 300°)
(20, 660°)
(20, -60°)

Part II Polar Coordinates to Rectangular Coordinates

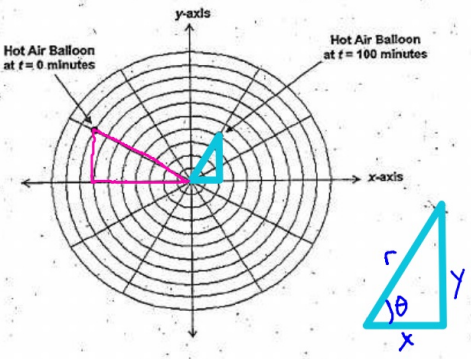
Although air traffic controllers work with radar screens that require using polar coordinates to locate aircraft, it is important for controllers to translate a location, shown in polar coordinates (r, θ) on their radar screens, into rectangular coordinates. The rectangular coordinates will be referenced to the standard directions of north-south (y-axis directions) and east-west (x-axis directions). In addition, air traffic controllers must also be able to translate an aircraft's location, given in rectangular coordinates, to a polar location on the polar grid.

Air Traffic Controller

STUDENT ACTIVITY (continued)

One afternoon, a hot air balloon suddenly appeared as a blip on the air traffic controller's radar screen. To air traffic controllers, hot air balloons can mean trouble, especially if they drift too close to an airport and the airport's airline traffic. In fact, hot air balloons are prohibited from flying within a 20-mile radius of the airport and its air traffic control tower.

9. The polar grid for the radar screen below shows the air traffic controller tracking a hot air balloon. The polar grid shows the blip for the hot air balloon at time $t = 0$ minutes and at a time 100 minutes later.



a. Express the two locations of the hot air balloon in polar coordinates.

b. Is the hot air balloon in violation of the airport air space at either of these locations? Explain below.

Air Traffic Controller

STUDENT ACTIVITY (continued)

Although the hot air balloon is not in violation of the 20-mile airport air space limit, the air traffic controller is obligated to warn the hot air balloon pilot of its possible encroachment of the airport air space. Since the hot air balloon pilot is not carrying a radio, the air traffic controller sends a police car to establish visual contact with the pilot to alert him or her of the possible danger. When visual contact is made by using the police car's warning lights and siren, the police officers will then use their amplified speakers to convey the warning to the balloonist. Rectangular north-south-east-west directions, referenced from the control tower, namely, rectangular coordinates with the origin placed at the pole, can be used to convey the exact location of the hot air balloon to the police.

10. Notice that the rectangular xy-coordinate system, its origin placed at the pole and the positive x-axis along the polar axis, has been superimposed on the polar grid in Question 9.

a. Use right triangle trigonometry to determine the rectangular coordinates for the hot air balloon's location when $t = 100$ minutes. Show work and explain your reasoning.

$\cos(60) = \frac{x}{20}$

$20\cos(60) = x$

b. How many miles east and how many miles north of the air traffic control tower is the hot air balloon at $t = 100$ minutes?

10 miles east

17.3 miles north

c. What are the rectangular coordinates for the location of the hot air balloon at $t = 0$ minutes?