Friday, August 30

*Chairs up at the end of class

Explain how to write an equation of a line, if you know the slope and one point the line passes through.

$y = mx + b$

slope = 5

point = (-7, 1)

$y - y_1 = m(x - x_1)$

$y - 1 = 5(x - (-7))$

$y - 1 = 5(x + 7)$

August 30 - Day 3

Students will verbally explain how to graph polar functions

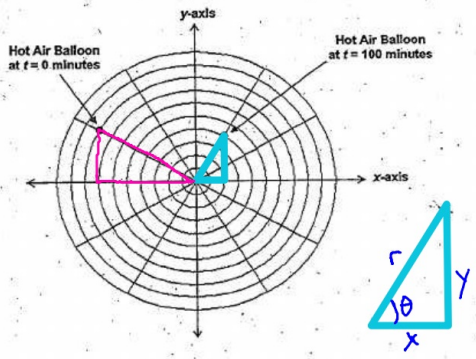
(using the words: angle, radius, terminal, initial...)

Air Traffic Controller

STUDENT ACTIVITY (continued)

One afternoon, a hot air balloon suddenly appeared as a blip on the air traffic controller's radar screen. To air traffic controllers, hot air balloons can mean trouble, especially if they drift too close to an airport and the airport's airline traffic. In fact, hot air balloons are prohibited from flying within a 20-mile radius of the airport and its air traffic control tower.

9. The polar grid for the radar screen below shows the air traffic controller tracking a hot air balloon. The polar grid shows the blip for the hot air balloon at time $t = 0$ minutes and at a time 100 minutes later.



a. Express the two locations of the hot air balloon in polar coordinates.

b. Is the hot air balloon in violation of the airport air space at either of these locations? Explain below.

Air Traffic Controller

STUDENT ACTIVITY (continued)

Although the hot air balloon is not in violation of the 20-mile airport air space limit, the air traffic controller is obligated to warn the hot air balloon pilot of its possible encroachment of the airport air space. Since the hot air balloon pilot is not carrying a radio, the air traffic controller sends a police car to establish visual contact with the pilot to alert him or her of the possible danger. When visual contact is made by using the police car's warning lights and siren, the police officers will then use their amplified speakers to convey the warning to the balloonist. Rectangular north-south-east-west directions, referenced from the control tower, namely, rectangular coordinates with the origin placed at the pole, can be used to convey the exact location of the hot air balloon to the police.

10. Notice that the rectangular xy-coordinate system, its origin placed at the pole and the positive x-axis along the polar axis, has been superimposed on the polar grid in Question 9.

a. Use right triangle trigonometry to determine the rectangular coordinates for the hot air balloon's location when $t = 100$ minutes. Show work and explain your reasoning.

$\cos(60) = \frac{x}{20}$

$20\cos(60) = x$

b. How many miles east and how many miles north of the air traffic control tower is the hot air balloon at $t = 100$ minutes?

10 miles east

17.3 miles north

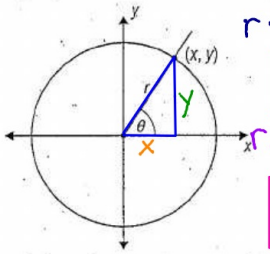
c. What are the rectangular coordinates for the location of the hot air balloon at $t = 0$ minutes?

Air Traffic Controller

STUDENT ACTIVITY (continued)

Air traffic controllers must be prepared for emergencies that require working with area police and fire services. Since fire and police agencies work with rectangular coordinates, generalized expressions translating (r, θ) coordinates into (x, y) coordinates are needed.

11. Given a point described by the polar pair (r, θ), use the diagram below and trigonometry to express the rectangular coordinates of x and y in terms of r and θ. Show work to support your answer.



$r \cdot \cos\theta = \frac{x}{r} \cdot r$

$r\cos\theta = x$

$r \cdot \sin\theta = \frac{y}{r} \cdot r$

$r\sin\theta = y$

12. Apply the results in Question 11 to write the rectangular coordinates (x, y) for the following polar coordinates (r, θ).

a. The rectangular coordinate pair for (20, 150°) is:

$x = 20\cos(150)$ $\quad y = 20\sin(150°)$

b. The rectangular coordinate pair for (12, 225°) is:

c. The rectangular coordinate pair for (10, –30°) is:

d. The rectangular coordinate pair for (–30, –315°) is:

e. The rectangular coordinate pair for (17, 128°) is:

Air Traffic Controller

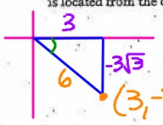
STUDENT ACTIVITY (continued)

Part III Translating from Rectangular to Polar

By the time the police are able to catch up to the hot air balloon, it is located 3 miles east and $3\sqrt{3}$ miles south of the control tower. The balloon is on the ground but not yet secured. The police report this location to the control tower and the air traffic controller records it in rectangular form as $(3, -3\sqrt{3})$. Because the balloon is on the ground, radar is unable to detect the balloon. Until the balloon is secured, it continues to pose a danger to air traffic in the area, especially if a gust of wind should suddenly send it airborne once again.

13. The air traffic controller must mark the balloon's location on the radar screen.

a. Based on the rectangular coordinates for the hot air balloon, determine if the balloon's position violated the airport's 20-mile limit by finding how far the balloon is located from the control tower. Show work to support your answer.



$r = 6$

b. What angle θ could be used to locate the hot air balloon on the polar grid? Show work to support your answer.

$\tan\theta = \frac{-3\sqrt{3}}{3}$

$\theta = \tan^{-1}\left(\frac{3\sqrt{3}}{3}\right) = -60$

c. Give a list of four polar coordinate pairs (r, θ) that could be used to locate the hot air balloon's position.

$(6, -60°)$ $\quad (6, 300°)$

$(6, 120°)$ $\quad (6, -420°)$

Air Traffic Controller

STUDENT ACTIVITY (continued)

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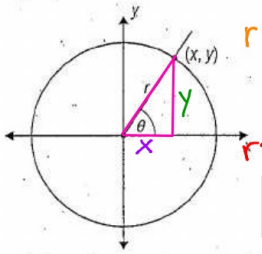

$r \cdot \cos\theta = \frac{x}{r} \cdot r$

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Air Traffic Controller

STUDENT ACTIVITY (continued)

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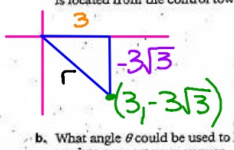


$3^2 + (-3\sqrt{3})^2 = r^2$

$9 + 27 = r^2$

$36 = r^2$ $\quad r = 6$

b. What angle θ could be used to locate the hot air balloon on the polar grid? Show work to support your answer.

$\cos\theta = \frac{3}{6}$

$\theta = \cos^{-1}\left(\frac{3}{6}\right) = 60°$ $\quad \theta = -60°$

c. Give a list of four polar coordinate pairs (r, θ) that could be used to locate the hot air balloon's position.

$(6, -60°)$

Air Traffic Controller

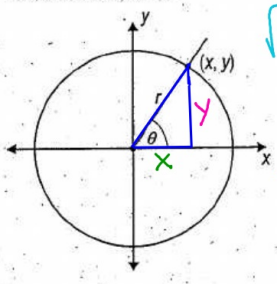
STUDENT ACTIVITY (continued)

14. Suppose that before the hot air balloon could be secured, a strong gust of wind pushed the balloon to a new location. When the balloon was finally secured, its new position in relation to the control tower was $(-2\sqrt{3}, -2)$.

a. How far from the control tower is the hot air balloon at this new position? Show work to support your answer.

b. Give two possible angles for θ that could be used for the hot air balloon's polar coordinates (r, θ). Show work to support your answer.

15. Given a point described by the rectangular pair (x, y), use the diagram below, and trigonometry, to express the polar coordinates of r and θ in terms of x and y. Show work to support your answer.



$\sqrt{x^2 + y^2} = \sqrt{r^2}$

$\sqrt{x^2 + y^2} = r$

$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Air Traffic Controller

STUDENT ACTIVITY (continued)

16. An approaching aircraft is 9 miles west and $9\sqrt{3}$ miles north of the air traffic control tower.

a. Determine the rectangular coordinates of the aircraft.

b. Use your results in Question 15 to determine the polar coordinates for this aircraft.

c. Calculators have not only trigonometric functions, but also inverse trigonometric functions. When $\tan^{-1}$ on a calculator is used to compute the angle for the aircraft at location $(-9, 9\sqrt{3})$, explain why the calculator gives the angle for the aircraft as located at $(9, -9\sqrt{3})$.

17. The rectangular coordinates of an approaching aircraft are given as $(-12, -9)$. Determine the polar coordinates for this aircraft.