

November 20

Using the sum and difference formula  
and your unit circle

(but no calculator).

Find the values of  $\sin(15)$  and  $\cos(15)$ .

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

(Hint:  $15 = 45 - 30$ )

$$\begin{aligned}\sin(15) &= \sin(45 - 30) \\ &= \sin(45) \cos(30) - \cos(45) \sin(30) \\ &= \frac{\sqrt{2}}{2} \left( \frac{\sqrt{3}}{2} \right) - \frac{\sqrt{2}}{2} \left( \frac{1}{2} \right) \\ &= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

$$\begin{aligned}\cos(15) &= \cos(45 - 30) \\ &= \cos(45) \cos(30) + \sin(45) \sin(30) \\ &= \frac{\sqrt{2}}{2} \left( \frac{\sqrt{3}}{2} \right) + \frac{\sqrt{2}}{2} \left( \frac{1}{2} \right) \\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}\end{aligned}$$



November 20

Students will verbally explain how to  
use the double angle and half angle  
formulas

(using the words:  
identity, reciprocal, quotient ...)



### Double-Angle Formulas:

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\cos 2u = 1 - \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Using the double angle formula, which expression is equivalent to  $2\cos(2x)$ ?

☐  $2\sin(x)\cos(x)$

☐  $4\sin(x)\cos(x)$

☐  $2(\cos^2 x - 1)$

☐  $4\cos(x)$

$$\cos 2u = 2 \cos^2 u - 1$$

$$\begin{aligned} &2(\cos 2x) \\ &2(2\cos^2 x - 1) \\ &4\cos^2 x - 2 \end{aligned}$$

### Double-Angle Formulas:

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\cos 2u = 1 - \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\cos 2u = 2 \cos^2 u - 1$$

Using the double angle formula, which expression is equivalent to  $2\sin(x)\cos(x)+2$ ?

☐  $\sin(2x)+1$

☐  $\sin(2x)+2$

☐  $\cos(2x)+1$

☐  $\cos(2x)+2$

$$2\sin x \cos x + 2$$

$$\sin 2u = 2 \sin u \cos u$$

$$\sin 2x + 2$$

### Double-Angle Formulas:

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\cos 2u = 1 - \sin^2 u$$

$$\cos 2u = 2 \cos^2 u - 1$$

Using the double angle formula, which expression is equivalent to  $7 \tan(2x)$ ?

☐  $\frac{14 \tan(x)}{1 - \tan^2(x)}$

☐  $\frac{14 \tan(x)}{1 + \tan^2(x)}$

☐  $\frac{7 \tan(x)}{1 - \tan^2(x)}$

☐  $\frac{7 \tan(x)}{1 + \tan^2(x)}$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$7(\tan 2x) = 7\left(\frac{2 \tan x}{1 - \tan^2 x}\right) = \frac{14 \tan x}{1 - \tan^2 x}$$

### Double-Angle Formulas:

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\cos 2u = 1 - \sin^2 u$$

$$\cos 2u = 2 \cos^2 u - 1$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Using the double angle formula, which expression is equivalent to  $4 \sin(x) \cos(x)$ ?

$$\sin 2u = 2 \sin u \cos u$$

$$4 \sin(x) \cos(x) = 2(2 \sin x \cos x) = 2(\sin 2x)$$

$$\cos\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{\sin u}{1 + \cos u}$$

Using the half angle formula, which expression is equivalent to  $\sin(75)$ ?

- ☐  $\pm \sqrt{\frac{1 - \cos(75)}{2}}$
- ☐  $\pm \sqrt{\frac{1 - \cos(150)}{2}}$
- ☐  $\pm \sqrt{\frac{1 + \cos(75)}{2}}$
- ☐  $\pm \sqrt{\frac{1 + \cos(150)}{2}}$

$$75 = \frac{u}{2}$$

$$2(75) = \frac{u}{2} \cdot 2$$

$$150 = u$$

$$\sin\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\sin(75)$$

$$\sin\left(\frac{150}{2}\right) = \pm \sqrt{\frac{1 - \cos(150)}{2}}$$

$$\sin\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{\sin u}{1 + \cos u}$$

Using the half angle formula, which expression is equivalent to  $\cos\left(\frac{11\pi}{12}\right)$ ?

- ☐  $\pm \sqrt{\frac{1 - \cos\left(\frac{11\pi}{12}\right)}{2}}$
- ☐  $\pm \sqrt{\frac{1 + \cos\left(\frac{11\pi}{12}\right)}{2}}$
- ☐  $\pm \sqrt{\frac{1 - \cos\left(\frac{11\pi}{6}\right)}{2}}$
- ☐  $\pm \sqrt{\frac{1 + \cos\left(\frac{11\pi}{6}\right)}{2}}$

$$2 \cdot \frac{11\pi}{12} = \frac{u}{2} \cdot 2$$

$$\frac{22\pi}{12} = u$$

$$\frac{11\pi}{6} = u$$

$$\sin\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{\sin u}{1 + \cos u}$$

Using the half angle formula, which expression is equivalent to

$$\sqrt{\frac{1 + \cos(4x)}{2}} ?$$

- ☐  $\cos(2x)$
- ☐  $\cos(4x)$
- ☐  $\sin(2x)$
- ☐  $\sin(4x)$

$$u = 4x$$

$$\cos\left(\frac{4x}{2}\right) = \cos(2x)$$

$$\cos\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\cos\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{\sin u}{1 + \cos u}$$

Using the half angle formula, write an expression is equivalent to

$$\sqrt{\frac{1 - \cos(x-1)}{2}} .$$

$$u = x - 1$$

$$\sin\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\sin\left(\frac{x-1}{2}\right)$$

pg 227 # 1 - 10 AND

Pick one:

#11 – 31 (odd)

OR

#11 – 44 (every 3<sup>rd</sup> problem)

pg 247 # 21 - 30

Pg 257 # 19 - 22, 29 - 34,  
53 - 56, 61 - 80