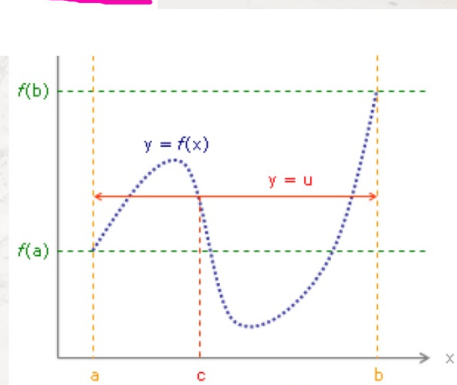
February 11

In your own words, what does the
INTERMEDIATE VALUE THEOREM say?

Formal Theorem:

If a function f is continuous on a closed interval $[a, b]$ and $f(a) \neq f(b)$, then for every value of u between $f(a)$ and $f(b)$, there exist at least one value of c in the open interval (a, b) so that $f(c) = u$.



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Students will verbally explain how to apply the Intermediate Value Theorem to Justify solutions

(using the words:
value, continuous, exist...)

Show that $g(t) = t^7 + 3t - 10$ equals zero on the interval [1, 2]

$g(1) = 1^7 + 3(1) - 10$
$= 1 + 3 - 10 = -6$

$g(2) = 2^7 + 3(2) - 10$
$= 128 + 6 - 10 = 124$

Since $g(t)$ is continuous
$g(1) = -6$ and $g(2) = 124$
$g(t) = 0$ between $t=1$ and $t=2$
by IVT

Show that 1/4 is a solution for $f(x) = x(\sin x)$

$f(5) = 5(\sin(5)) = 5\sin(5)$

$f(1) = 1(\sin(1)) = \sin(1)$

$f(\pi) = \pi(\sin(\pi)) = \pi(0) = 0 < \frac{1}{4}$

$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\left(\sin\left(\frac{\pi}{2}\right)\right) = \frac{\pi}{2}(1) = \frac{\pi}{2} > \frac{1}{4}$

Since $f(x) = x\sin x$ is continuous
$f(\pi) = 0$ and $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$
$f(x) = \frac{1}{4}$ between $x = \frac{\pi}{2}$ and $x = \pi$
by IVT

Show that 1/4 is a solution for $f(x) = x(\sin x)$

$f(1) = 1(\sin(1)) = \sin(1)$

$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\left(\sin\left(\frac{\pi}{2}\right)\right) = \frac{\pi}{2}(1) = \frac{\pi}{2} > \frac{1}{4}$

$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}\left(\sin\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}\left(\frac{1}{2}\right) = \frac{\pi}{12} > \frac{1}{4}$

$f(0) = 0(\sin(0)) = 0(0) = 0 < \frac{1}{4}$

Since $f(x) = x\sin x$ is continuous

$f(0) = 0$ and $f\left(\frac{\pi}{6}\right) = \frac{\pi}{12}$

$f(x) = \frac{1}{4}$ between $x = 0$ and $x = \frac{\pi}{6}$

by IVT

Sketch a graph with infinite one-sided limits at x = 0 that does not satisfy the conclusion of the IVT

(y-values exist)