February 21

How is finding the slope of the secant line different from finding the slope of the tangent line?

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Students will verbally explain how to find the slope of the tangent line

(using the words:
limit, secant, tangent...)

Slope of
Secant Line

$$\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$(a, f(a))$ $(b, f(b))$

$$\boxed{\frac{f(b)-f(a)}{b-a}}$$

Slope of
Tangent Line

$$\lim_{b \to a} \frac{f(b)-f(a)}{b-a}$$

Equation 1

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{x+h-x} = \boxed{\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}}$$

Slope of tangent line

Find the slope of the line tangent to $f(x) = x^2$ at $x = 1$

2 points

$(1, 1)$ $(1+h, (1+h)^2)$

p

$$\text{Slope of secant line} = \frac{(1+h)^2 - 1}{(1+h) - 1} = \frac{(1+h)(1+h) - 1}{1+h-1}$$

$$= \frac{1+2h+h^2-1}{1+h-1} = \frac{2h+h^2}{h}$$

$$= \frac{h(2+h)}{h} = 2+h$$

$$\text{Slope of tangent line} = \lim_{h \to 0} \frac{(1+h)^2 - 1}{(1+h) - 1} = \lim_{h \to 0} 2+h = 2$$

Write an equation of the line tangent to $f(x) = x^2$ at $x = 1$

slope $= 2$

point $= (1, 1)$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 2(x - 1)$$

Find the slope of the line tangent to $f(x) = x^2$ at $x = 2$

2 points

$(2, 4)$ $\quad (2+h, (2+h)^2)$

2^2

$$\text{Slope of secant line} = \frac{(2+h)^2 - 4}{(2+h) - 2} = \frac{(2+h)(2+h) - 4}{2+h-2}$$

$$= \frac{4+4h+h^2-4}{2+h-2} = \frac{4h+h^2}{h}$$

$$= \frac{h(4+h)}{h} = 4+h$$

$$\text{Slope of tangent line} = \lim_{h \to 0} \frac{(2+h)^2 - 4}{(2+h) - 2} = \lim_{h \to 0} 4+h = 4$$

Write an equation of the line tangent to $f(x) = x^2$ at $x = 2$

slope = 4

point = (2, 4)

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 4(x - 2)$$

Find the slope of the line tangent to $f(x) = x^2$ at any x-value

2 points

(x, x^2) $\quad (x+h, (x+h)^2)$

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{x+h-x}$$

$$= \lim_{h \to 0} \frac{(x+h)(x+h) - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2+2xh+h^2-x}{h} = \lim_{h \to 0} \frac{2xh+h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h} = \lim_{h \to 0} 2x+h = 2x$$

Slope of tangent line at any x-value is $2x$

$x = 10 \longrightarrow$ slope $= 2(10) = 20$

$x = -7 \longrightarrow$ slope $= 2(-7) = -14$

Derivative

tells you the slope of the tangent line at any x-value

→ instantaneous rate of change

Derivative Notation

$f'(x) \rightarrow$ "f prime of x"

$f(x) = x^2 \rightarrow f'(x) = 2x$

$y' \rightarrow$ "y prime"

$y = x^2 \rightarrow y' = 2x$

$\frac{dy}{dx} \rightarrow$ "d-y over d-x"

the derivative of y with respect to x

$y = x^2 \rightarrow \frac{dy}{dx} = 2x$

$\frac{d}{dx}(\ \ ) \rightarrow$ derivative of ( ) with respect to x

$\frac{d}{dx}(x^2) = 2x$

Definition of the Derivative

$$f'(x) = \boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$y = x^2 - 7x + 20$

Find $\frac{dy}{dx}$

$(y', f'(x))$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = x^2 - 7x + 20$$

$$f(x+h) = (x+h)^2 - 7(x+h) + 20$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 7(x+h) + 20 - (x^2 - 7x + 20)}{h}$$