

March 17

When do you need to take the derivative of a function?

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Students will verbally explain how to find the rate of change  
(using the words:  
derivative, average, instantaneous...)

Average  
Rate of Change

slope of  
secant line

Algebra I  
slope

$$\frac{f(b) - f(a)}{b - a}$$

Instantaneous  
Rate of Change

$$f'(x)$$

slope of  
tangent line

Calculus  
Slope

Derivative

Find the average rate of change for the area of a circle with respect to its radius when  $r = 1$  to  $r = 4$

average rate of change  $\rightarrow$  2 points

$$\frac{\Delta A}{\Delta r}$$

$$A = \pi r^2$$

$$r = 1$$

$$A = \pi(1)^2 = \pi$$

$$r = 4$$

$$A = \pi(4)^2 = 16\pi$$

$$\frac{\Delta A}{\Delta r} = \frac{16\pi - \pi}{4 - 1} = \frac{15\pi}{3} = \boxed{5\pi}$$

Find the instantaneous rate of change for the area of a circle with respect to its radius when  $r = 3$

inst. rate of change  $\rightarrow$  derivative of  $A = \pi r^2$  at  $r = 3$

$$\frac{dA}{dr} = \pi(2r) = 2\pi r$$

$$\text{at } r = 3 \quad \frac{dA}{dr} = 2\pi(3) = \boxed{6\pi}$$

In Exercises 1–8, find the rate of change.

1. Area of a square with respect to its side  $s$  when  $s = 5$ .

Show Answer

instantaneous

$$A = bh = b \cdot b = b^2$$

$$A = s^2$$

2. Volume of a cube with respect to its side  $s$  when  $s = 5$ .

3. Cube root  $\sqrt[3]{x}$  with respect to  $x$  when  $x = 1, 8, 27$ .

Show Answer

$$\sqrt[3]{x} = x^{1/3}$$

inst. at  $x=1$

$$x=8$$

and

$$x=27$$

4. The reciprocal  $1/x$  with respect to  $x$  when  $x = 1, 2, 3$ .

$$\frac{1}{x} = x^{-1}$$

5. The diameter of a circle with respect to radius.

Show Answer

6. Surface area  $A$  of a sphere with respect to radius  $r$  ( $A = 4\pi r^2$ ).

7. Volume  $V$  of a cylinder with respect to radius if the height is equal to the radius.