

A photograph of a green and yellow train stopped at a station platform. The train is facing the viewer, and the platform is visible on the left. The background shows a building and a cloudy sky.

March 21

How do you determine if an object is moving right or left?

A photograph of a green and yellow train stopped at a station platform. The train is facing the viewer, and the platform is visible on the left. The background shows a building and a cloudy sky.

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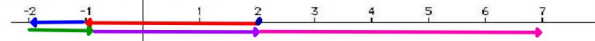
Students will verbally explain how to find and interpret velocity and acceleration functions
(using the words:
rate of change, derivative...)

Example 2) Given that a particle is moving along a horizontal line with position function $s(t) = t^2 - 4t + 2$.

The velocity function $v(t) = 2t - 4$ and the acceleration function $a(t) = 2$.

Let's complete the chart for the first 5 seconds and show where the object is on the number line.

t	s(t)	v(t)	v(t)	a(t)	Description of the particle's motion
0	2	-4	4	2	moving left, positive acceleration
1	-1	-2	2	2	moving left, positive acceleration
2	-2	0	0	2	not moving, positive acceleration
3	-1	2	2	2	moving right, positive acceleration
4	2	4	4	2	moving right, positive acceleration
5	7	6	6	2	moving right, positive acceleration



It is too much work to do such work for complicated functions. We are generally interested when the particle is stopped or when it has no acceleration. We are also interested when the object is speeding up or slowing down. Realizing that an object's velocity is either, positive (moving right), negative (moving left) or zero (stopped) and an object's acceleration is either positive, negative, or zero (constant speed), we can now use a chart to determine all the possibilities of an object's motion as if you were looking at it from above.

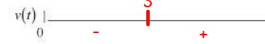
	a(t) > 0	a(t) < 0	a(t) = 0
v(t) > 0	right, positive acc.	right, negative acc.	right, constant speed
v(t) < 0	left, positive acc.	left, negative acc.	left, constant speed
v(t) = 0	stopped, positive acc.	stopped, negative acc.	stopped, no acceleration

Example 3) A particle is moving along a horizontal line with position function $s(t) = t^2 - 6t + 5$. Do an analysis

of the particle's direction (right, left), acceleration, motion (speeding up, slowing down), & position.

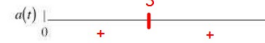
Step 1: $v(t) = 2t - 6$. So $v(t) = 0$ at $t = 3$.

Step 2: Make a number line of $v(t)$ showing when the object is stopped and the sign and direction of the object at times to the left and right of that. Assume $t \geq 0$.



Step 3: $a(t) = 2$. Does $a(t) = 0$? no.

Step 4: Make a number line of $a(t)$ showing when the object has a positive and negative acceleration. Scale it exactly like the $v(t)$ number line.



Step 5: Make a motion line directly below the last two putting all critical values, multiplying the signs and interpreting according to the chart above.



Step 6: Make a position graph to show where the object is at critical times and how it moves.



The position of a particle is given by

$$s(t) = 4t^3 - 10t^2 + 8t - 13$$

Determine when the particle is moving right and left.

① find velocity function

$$s'(t) = v(t) = 12t^2 - 20t + 8$$

② set $v(t) = 0$ and solve

$$0 = 12t^2 - 20t + 8$$

$$0 = 4(3t^2 - 5t + 2)$$

$$0 = 4(3t^2 - 3t - 2t + 2)$$

$$0 = 4[3t(t-1) - 2(t-2)]$$

$$0 = 4(t-1)(3t-2)$$

$$0 \neq 4 \quad 0 = t-1 \quad 0 = 3t-2$$

$$1 = t \quad 2 = 3t$$

$$\frac{2}{3} = t$$

Critical points
(where the particle is stopped)

③ create a sign chart to determine direction

	$\frac{2}{3}$	1	
critical points			
sign $v(t)$	+	-	+
behavior of particle	right	left	right

$$v(t) = 4(t-1)(3t-2)$$

$$\checkmark a \neq \text{less than } \frac{2}{3}$$

$$v(\frac{1}{3}) = 4(-)(-) = +$$

$$\checkmark a \neq \text{greater than } 1$$

$$v(1.5) = 4(+)(+) = +$$

$$\checkmark a \neq \text{between } \frac{2}{3} \text{ and } 1$$

$$v(.8) = 4(-)(+) = -$$

④ interpret sign chart

moving right when $t < \frac{2}{3}$, $t > 1$

moving left when $\frac{2}{3} < t < 1$

Given the position of the particle, determine when the particle is moving right and left.

$$s(t) = -2x^3 + 6x^2 - 3$$

$$v(x) = -6x^2 + 12x$$

$$0 = -6x^2 + 12x$$

$$0 = -6x(x - 2)$$

$$x = 0, 2$$

	0		2	
stop				
sign v	-	+	-	
direction	left	right	left	