May 14, 2014

Solve the following equations:

$2\cos(x)+1 = 0$

$-2\cos(x) \quad -2\cos(x)$

$\frac{1}{-2} = \frac{-2\cos(x)}{-2}$

$-\frac{1}{2} = \cos(x)$

$\cos^{-1}\left(-\frac{1}{2}\right) = x$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3}+2\pi n, \frac{4\pi}{3}+2\pi n$

$\frac{3\tan^2(x)}{3} = \frac{3}{3}$

$\tan^2(x) = 1$

$\tan(x) = \pm 1$

$x = \tan^{-1}(\pm 1)$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\frac{(2n+1)\pi}{4}$

May 14

Students will verbally explain how to determine properties of the function, its derivative and its second derivative.

(using the words:
positive, negative, increasing, decreasing, concave up, concave down, etc...)

Important Things to Know:

Project is due TUESDAY - MAY 20th

TODAY is the last day to take your test

This week you can make-up one of each:

* multiple choice
* free response
* quiz

Quiz THURSDAY - open notes

* derivatives
* implicit differentiation
* related rates
* completing the square, solving (logs, exponentials, trig functions)

Conjecture 1: Write a statement that relates the zeros of the derivative and the location of the max/min of the function.

The zeros of the derivative are the x-values of the maximum or minimum of the function

Conjecture 2: Write a statement that relates the second derivative and the concavity of a function.

If the sign of the second derivative is positive, the function is concave ~~down~~ up.
If the sign of the second derivative is negative, the function is concave ~~up~~. down

Conjecture 3: Write a statement that relates the sign of the second derivative and whether the zero of the first derivative is a max/min of the function.

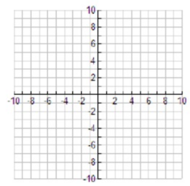
If the sign of the second derivative is positive, the function has a minimum.
If the sign of the second derivative is negative, the function has a maximum.

Find the vertex and concavity of the following functions using what you have learned about the first and second derivatives.

Function	$\frac{dy}{dx}$	zero of $\frac{dy}{dx}$	Vertex of y	$\frac{d^2y}{dx^2}$	Sign of $\frac{d^2y}{dx^2}$	Vertex of y is max/min
E. $y = x^2 - 3x$						
F $y = -2x^2 + 8x + 1$						

II. Increasing/Decreasing Functions

Example A. Consider $y = x^2 - 5x - 1$. Use the graph of the function to determine the following; you may use the max/min function on the calculate menu to speed the process. Graph and label y on the grid below.



For which interval(s) is y increasing (use x-values to write intervals)? _______________

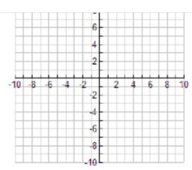
For which interval(s) is y decreasing (use x-values to write intervals)? _______________

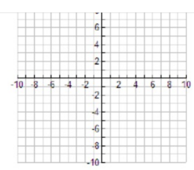
Find $\frac{dy}{dx}$: ____________________ Graph and label it on the grid above.

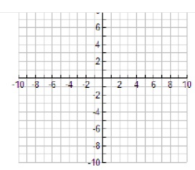
What is the sign of $\frac{dy}{dx}$ on the interval(s) for which y is increasing? _______________

What is the sign of $\frac{dy}{dx}$ on the interval(s) for which y is decreasing? _______________

Note--I am asking for the sign of the y-values on a particular intervals of x-values: choose an x-value in the interval and replace that value for x in the derivative to determine the sign of y-values of the derivative for that interval of x-values or use the graph of the derivative to determine the sign.







Conjecture 4: Write a statement that relates the sign of the first derivative and whether the function increases or decreases.

Create a sign chart using the critical values of $\frac{dy}{dx}$ (where $\frac{dy}{dx}=0$ or where $\frac{dy}{dx}$ is undefined) and what you have learned about the sign of the first derivative to determine the intervals for which the following functions are increasing/decreasing.

Function	$\frac{dy}{dx}$	Critical values of $\frac{dy}{dx}$	Interval(s) for which y is increasing	Interval(s) for which y is decreasing
E $y=x^2-6x+5$	$2x-6$	3	$x>3$ $(3,\infty)$	$x<3$ $(-\infty,3)$
Sign chart to determine sign of $\frac{dy}{dx}$: $2x-6=0$, $2x=6$, $x=3$				
F. $y=-x^2+x+1$				
Sign chart to determine sign of $\frac{dy}{dx}$				

CP		3	
Sign y'	$-$		$+$
Behav. y	dec		inc

III. Summary of Curve Sketching

An inflection point is a point on the graph where the function changes concavity. To find possible inflection points, find where $\frac{d^2y}{dx^2}=0$ or is undefined. Using this information and the conjectures you have made in this activity, analyze the following functions completely.

Function	$\frac{dy}{dx}$	Critical values of $\frac{dy}{dx}$	Interval(s) for which y is increasing	Interval(s) for which y is decreasing
1. $y=5+3x^2-x^3$	$y'=6x-3x^2$	0, 2	$0<x<2$	$x<0,\ x>2$
Sign chart to determine sign of $\frac{dy}{dx}$: $6x-3x^2=0$, $3x(2-x)=0$, $3x=0$, $x=0$; $2-x=0$, $x=2$				
	$\frac{d^2y}{dx^2}$	Critical values of $\frac{d^2y}{dx^2}$	Interval(s) for which y is concave up	Interval(s) for which y is concave down
	$y''=6-6x$	$x=1$	$x<1$	$x>1$
Sign chart to determine sign of $\frac{d^2y}{dx^2}$: $6-6x=0$, $6=6x$, $1=x$				

CP		0		2	
Sign y'	$-$		$+$		$-$
Behav. y	dec		inc		dec

possible inflection point (pip)		1	
Sign y''	$+$		$-$
Behav. y	concave up (cu)		concave down (cd)

Use the critical values of $\frac{dy}{dx}$ and their sign in $\frac{d^2y}{dx^2}$ to determine the relative max/min(s) of y:

$x=0 \rightarrow$ minimum because y'' is positive
$x=2 \rightarrow$ maximum because y'' is negative

Use the critical values of $\frac{d^2y}{dx^2}$ to determine any inflection points of y:

$x=1$ because y'' changes sign

Draw and label y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ on the grid.