November 15

SWBAT:

Use the sum and difference ~~and difference~~ formulas to simplify and evaluate trig expressions

Write an equation of a sine function with:

amplitude = 4

period = 15

horizontal shift = -3

vertical shift = 1

$$y = 4\sin\left(\frac{2\pi}{15}x + \frac{6\pi}{15}\right) + 1$$

$$15 = \frac{2\pi}{b}$$

$$-3 = \frac{-c}{b} \rightarrow -3b = -c$$

$$-3\left(\frac{2\pi}{15}\right) = -c$$

Write an equation of a sine function with:

amplitude = 10

period = 27

horizontal shift = 5

vertical shift = -3

$$y = 10\sin\left(\frac{2\pi x}{27} + \frac{-10\pi}{27}\right) - 3$$

Find:

sin(30) $= \frac{1}{2}$

sin(45) $= \frac{\sqrt{2}}{2}$

sin(75) $= \sin(30+45) = \frac{1}{2} + \frac{\sqrt{2}}{2} = \frac{1+\sqrt{2}}{2}$

$\sin(75) = .966 \neq 1.707 = \frac{1+\sqrt{2}}{2}$

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\sin(75) = \sin(30+45) = \sin(30)\cos(45) + \cos(30)\sin(45)$

$u = 30 \quad v = 45$

$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4} = .966$

Sum and Difference Formulas:

$\sin(u-v) = \sin u \cos v - \cos u \sin v$ $\quad \cos(u-v) = \cos u \cos v + \sin u \sin v$ $\quad \tan(u-v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

$\sin(u+v) = \sin u \cos v + \cos u \sin v$ $\quad \tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$

$\cos\left(\frac{7\pi}{12}\right)$

$\frac{7\pi}{12} = \frac{2\pi}{12} + \frac{5\pi}{12} = \frac{\pi}{6} + \frac{5\pi}{12}$ ← not on unit circle

$\frac{7\pi}{12} = \frac{3\pi}{12} + \frac{4\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$

$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$u = \frac{\pi}{4}$ $\quad v = \frac{\pi}{3}$ $\quad \cos(u+v) = \cos u \cos v - \sin u \sin v$

$\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$

$= \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) - \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$

Sum and Difference Formulas:

$\sin(u-v) = \sin u \cos v - \cos u \sin v$ $\quad \cos(u-v) = \cos u \cos v + \sin u \sin v$

$\sin(u+v) = \sin u \cos v + \cos u \sin v$ $\quad \cos(u+v) = \cos u \cos v - \sin u \sin v$ $\quad \tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$

$\tan\left(\frac{-\pi}{12}\right)$

$-\frac{\pi}{12} = \frac{3\pi}{12} - \frac{4\pi}{12} = \frac{\pi}{4} - \frac{\pi}{3}$

$\tan\left(\frac{-\pi}{12}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$ $\quad \tan(u-v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

$u = \frac{\pi}{4}$ $\quad v = \frac{\pi}{3}$

$\tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \dfrac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{3}\right)}{1 + \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)}$

$= \dfrac{1 - \sqrt{3}}{1 + (1)(\sqrt{3})} = \dfrac{1-\sqrt{3}}{1+\sqrt{3}}$

Sum and Difference Formulas:

$$\sin(u-v)=\sin u\cos v-\cos u\sin v \qquad \tan(u-v)=\frac{\tan u-\tan v}{1+\tan u\tan v}$$

$$\cos(u+v)=\cos u\cos v-\sin u\sin v$$

$$\tan(u+v)=\frac{\tan u+\tan v}{1-\tan u\tan v}$$

Rewrite:

$\sin(20)\cos(30)+\cos(20)\sin(30)$

$\sin(u+v)=\sin u\cos v+\cos u\sin v$

$u=20 \quad v=30$

$=\sin(20+30)$

$=\sin(50)$

$\cos(4x)\cos(x)+\sin(4x)\sin(x)$

$\cos(u-v)=\cos u\cos v+\sin u\sin v$

$u=4x \quad v=x$

$=\cos(4x-x)$

$=\cos(3x)$

Sum and Difference Formulas:

$$\sin(u-v)=\sin u\cos v-\cos u\sin v \qquad \cos(u-v)=\cos u\cos v+\sin u\sin v \qquad \tan(u-v)=\frac{\tan u-\tan v}{1+\tan u\tan v}$$

$$\sin(u+v)=\sin u\cos v+\cos u\sin v \qquad \cos(u+v)=\cos u\cos v-\sin u\sin v$$

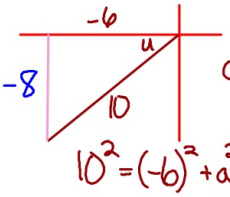
$\cos(u)=\frac{-6}{10}$

$\sin(v)=\frac{-12}{13}$

$\pi\le u\le\frac{3\pi}{2}$

$\pi\le v\le\frac{3\pi}{2}$

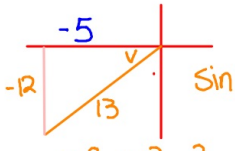
find $\tan(u+v)$



$\cos(u)=\frac{-6}{10}=\frac{adj}{hyp}$

$10^2=(-6)^2+a^2$

$\tan(u)=\frac{-8}{-6}=\frac{4}{3}$



$\sin(v)=\frac{-12}{13}=\frac{opp}{hyp}$

$13^2=12^2+a^2$

$\tan(v)=\frac{-12}{-5}=\frac{12}{5}$

$\tan(u+v)=\frac{\tan u+\tan v}{1-\tan u\tan v}$

$$\tan(u+v)=\frac{\tan(u)+\tan(v)}{1-\tan(u)\tan(v)}=\frac{\frac{4}{3}+\frac{12}{5}}{1-\frac{4}{3}\left(\frac{12}{5}\right)}$$

$$=\frac{\frac{20}{15}+\frac{36}{15}}{1-\frac{48}{15}}=\frac{\frac{56}{15}}{\frac{15}{15}-\frac{48}{15}}=\frac{\frac{56}{15}}{\frac{-33}{15}}$$

$$=\frac{56}{15}\cdot\frac{15}{-33}=-\frac{56}{33}$$