November 19

SWBAT:

Evaluate and simplify expressions using the power-reducing and half-angle formulas

$$\sin^2 u = \frac{1-\cos 2u}{2}$$

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

$$\cos^2 u = \frac{1+\cos 2u}{2}$$

Rewrite $\cos^2(3x)$ using the power reducing formula

$u = 3x$

$$\cos^2(3x) = \frac{1+\cos(2\cdot 3x)}{2}$$

$$= \frac{1+\cos(6x)}{2}$$

Rewrite $\sin^4 x$ using the power-reducing formulas

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

$$\sin^2 u = \frac{1-\cos 2u}{2}$$

$$\sin^4 x = (\sin^2 x)(\sin^2 x)$$

$$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{(1-\cos 2x)(1-\cos 2x)}{4}$$

$$= \frac{1-\cos 2x-\cos 2x+\cos^2(2x)}{4}$$

$$= \frac{1-2\cos(2x)+\cos^2(2x)}{4} \qquad \cos^2 u = \frac{1+\cos 2u}{2}$$

$u=2x$

$2(2x)$

$$= \frac{1-2\cos(2x)+\left(\frac{1+\cos(4x)}{2}\right)}{4}$$

evaluate $\sin(75)$ without a calculator using the half-angle formula

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos u}{2}} \qquad \tan\left(\frac{u}{2}\right) = \frac{\sin u}{1+\cos u}$$

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$75 = \frac{u}{2} \rightarrow 75(2) = 150 = u$$

$$\sin(75) = \sin\left(\frac{150}{2}\right) = \pm\sqrt{\frac{1-\cos(150)}{2}}$$

$$= \pm\sqrt{\frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{2}}$$

$$= \pm\sqrt{\frac{1}{2}\left(1+\frac{\sqrt{3}}{2}\right)} = \pm\sqrt{\frac{1}{2}+\frac{\sqrt{3}}{4}}$$

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \tan\left(\frac{u}{2}\right) = \frac{\sin u}{1+\cos u}$$

evaluate $\cos\left(\frac{11\pi}{12}\right)$

$$\frac{11\pi}{12} = \frac{u}{2} \rightarrow 2\left(\frac{11\pi}{12}\right) = \left(\frac{u}{2}\right)2 \rightarrow \frac{11\pi}{6} = u$$

$$\cos\left(\frac{11\pi}{12}\right) = \cos\left(\frac{\frac{11\pi}{6}}{2}\right)$$

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos u}{2}} \qquad = \pm\sqrt{\frac{1+\cos\left(\frac{11\pi}{6}\right)}{2}}$$

$$= \pm\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{1}{2}\left(1+\frac{\sqrt{3}}{2}\right)}$$

$$= \pm\sqrt{\frac{1}{2}+\frac{\sqrt{3}}{4}}$$

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}} \qquad \tan\left(\frac{u}{2}\right) = \frac{\sin u}{1+\cos u}$$

evaluate $\cos\left(\frac{11\pi}{12}\right)$

$$\frac{11\pi}{12} = \frac{u}{2} \rightarrow \frac{2}{1}\left(\frac{11\pi}{12}\right) = \left(\frac{u}{2}\right)\frac{2}{1} \rightarrow \frac{11\pi}{6} = u$$

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\cos\left(\frac{11\pi}{12}\right) = \cos\left(\frac{\frac{11\pi}{6}}{2}\right) = \pm\sqrt{\frac{1+\cos\left(\frac{11\pi}{6}\right)}{2}}$$

$$= \pm\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \pm\sqrt{\frac{1}{2}+\frac{\sqrt{3}}{4}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{\sin u}{1+\cos u}$$

Rewrite

$$\sqrt{\frac{1+\cos(4x)}{2}}$$

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos u}{2}}$$

$u = 4x$

$$\sqrt{\frac{1+\cos(4x)}{2}} = \cos\left(\frac{4x}{2}\right) = \cos(2x)$$

$$\sqrt{\frac{1-\cos(x-1)}{2}}$$

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}}$$

$u = x-1$

$$\sqrt{\frac{1-\cos(x-1)}{2}} = \sin\left(\frac{x-1}{2}\right)$$